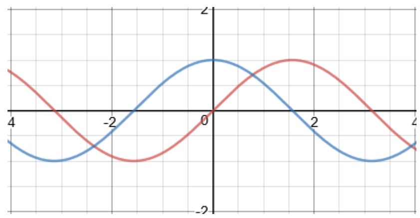
Review: Derivative of Trig Functions - 10/26/16

1 Derivative of Sine

The red is the graph of $\sin(x)$ and the blue is the graph of its derivative.



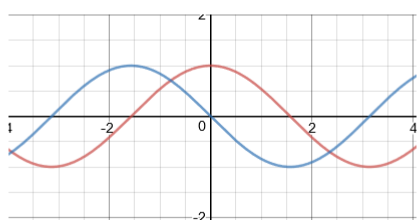
Derivative of $\sin$**:** $\frac{d}{dx}\sin(x) = \cos(x)$.

Example 1.0.1 *What is* $\frac{d}{dx}x^3\sin(x)$*? Let* $f(x) = x^3$ *and* $g(x) = \sin(x)$*, so* $f'(x) = 3x^2$ *and* $g'(x) = \cos(x)$*. Then by the product rule, we have* $\frac{d}{dx}x^3\sin(x) = 3x^2\sin(x) + x^3\cos(x)$.

Example 1.0.2 *What is* $\frac{d}{dx}\frac{3\sin(x)}{x^2}$*? Let* $f(x) = 3\sin(x)$ *and* $g(x) = x^2$*, so* $f'(x) = 3\cos(x)$ *and* $g'(x) = 2x$*. Then by the quotient rule, we have* $\frac{d}{dx}\frac{3\sin(x)}{x^2} = \frac{3\cos(x)x^2 - 2x(3\sin(x))}{x^4} = \frac{3x^2\cos(x) - 6x\sin(x)}{x^4}$.

2 Derivative of Cosine

The red is the graph of $\cos(x)$ and the blue is the graph of the derivative.



Derivative of $\cos$**:** $\frac{d}{dx}\cos(x) = -\sin(x)$.

Example 2.0.3 *What is* $\frac{d}{dx}\sin(x) + \cos(x)$*? It is* $\cos(x) - \sin(x)$.

Example 2.0.4 *What is* $\frac{d}{dx}\sin(x)\cos(x)$*? Let* $f(x) = \sin(x)$ *and* $g(x) = \cos(x)$*, so* $f'(x) = \cos(x)$ *and* $g'(x) = -\sin(x)$*. Then by the product rule, we have* $\cos(x)\cos(x) - \sin(x)\sin(x) = \cos^2(x) - \sin^2(x)$.

Example 2.0.5 *What is* $\frac{d^2}{dx^2}\sin(x)\cos(x)$*? We can just take the derivative of the derivative, which we found above, so* $\frac{d^2}{dx^2}\sin(x)\cos(x) = \frac{d}{dx}\cos^2(x) - \sin^2(x)$*. Let's do* $\frac{d}{dx}\cos^2(x)$ *first. We can do this using the product rule: let* $f(x) = \cos(x)$ *and* $g(x) = \cos(x)$*, so* $f'(x) = g'(x) = -\sin(x)$*. Then* $\frac{d}{dx}\cos^2(x) = -\sin(x)\cos(x) - \sin(x)\cos(x) = -2\sin(x)\cos(x)$*. Now let's do* $\frac{d}{dx}\sin^2(x)$*: let* $h(x) = k(x) = \sin(x)$*, so* $h'(x) = k'(x) = \cos(x)$*. Then* $\frac{d}{dx}\sin^2(x) = \cos(x)\sin(x) + \cos(x)\sin(x) = 2\cos(x)\sin(x)$*. Now we subtract them to get* $\frac{d^2}{dx^2}\sin(x)\cos(x) = \frac{d}{dx}\cos^2(x) - \sin^2(x) = -2\sin(x)\cos(x) - 2\cos(x)\sin(x) = -4\sin(x)\cos(x)$.

3 Derivative of Tangent

It turns out that looking at the graph for $\tan(x)$ is not incredibly illuminating. However, we can do it using algebra!

Example 3.0.6 *What is* $\frac{d}{dx}\tan(x)$*? This is the same as asking what is* $\frac{d}{dx}\frac{\sin(x)}{\cos(x)}$*. We can use the quotient rule here. Let* $f(x) = \sin(x)$ *and* $g(x) = \cos(x)$*, so* $f'(x) = \cos(x)$ *and* $g'(x) = -\sin(x)$*. Then* $\frac{d}{dx}\tan(x) = \frac{\cos(x)\cos(x)-(-\sin(x)\sin(x))}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$*.*

Derivative of $\tan$**:** $\frac{d}{dx}\tan(x) = \sec^2(x)$.

Example 3.0.7 *What is* $\frac{d}{dx}\frac{\tan(x)}{x}$*? Let* $f(x) = \tan(x)$ *and* $g(x) = x$*, then* $f'(x) = \sec^2(x)$ *and* $g'(x) = 1$*. Then by the quotient rule, I have* $\frac{d}{dx}\frac{\tan(x)}{x} = \frac{x\sec^2(x)-\tan(x)}{x^2}$*.*

Example 3.0.8 *What is* $\frac{d}{dx}\cot(x)$*? We can think of this as* $\frac{d}{dx}\frac{1}{tan(x)}$*. Let* $f(x) = 1$ *and* $g(x) = \tan(x)$*, so* $f'(x) = 0$ *and* $g'(x) = \sec^2(x)$*. Then by the quotient rule, we have*

$$\frac{d}{dx}\cot(x) = \frac{-\sec^2(x)}{\tan^2(x)} = \frac{-\frac{1}{\cos^2(x)}}{\frac{\sin^2(x)}{\cos^2(x)}} = -\frac{1}{\sin^2(x)} = -\csc^2(x).$$

Practice Problems
Look at the problems and solutions that we did in class.